Modeling and Analysis of Single Inverted Pendulum System

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Abstract: Single inverted pendulum system is the most common experimental object in University laboratories, especially for students and teachers of control specialty. Starting with its mechanical structure and characteristic parameters, according to its physical characteristic parameters, this paper analyses the relationship between force and motion, lists the kinematics equation of inverted pendulum system according to Newton's law of mechanics, establishes its system model, and provides the basis for subsequent control and digital simulation analysis.

1. Introduction

Inverted pendulum system is a high-order, unstable, multi-variable, non-linear and strongly coupled system as figure 1. Because it is similar to human standing on one foot in structure, it is the simplest model of many control objects, such as biped walking robot and rocket vertical attitude. Therefore, it has very important significance for the development and research of biped robot.

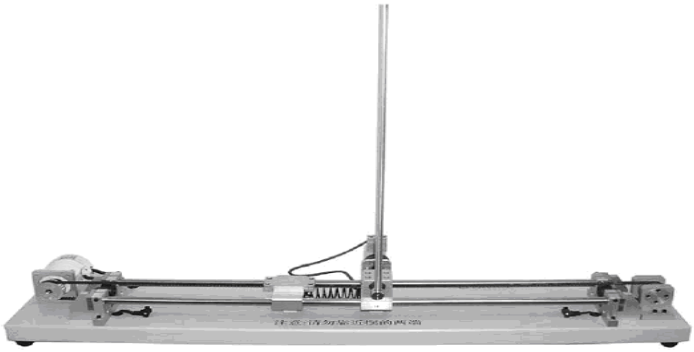

Fig.1 Linear inverted pendulum

The research of inverted pendulum system began in 1950s [1-5]. In the early stage, it mainly focused on the modeling of linear inverted pendulum and the balance control of pendulum rod (so-called stabilization problem) [6-10]. With the development of modern control theory, especially multivariable linear system theory and optimal theory, the research on inverted pendulum system has attracted more attention since 1970s[11-14]. In the late 1980s, with the rapid development of fuzzy control theory, the use of fuzzy control theory to control inverted pendulum has attracted extensive attention. Since the early 1990s, the research on neural network control of inverted pendulum has developed rapidly. Neural network control of inverted pendulum is based on self-learning and uses a new concept to process information, showing great potential. Great progress has been made in the application of neural network to inverted pendulum control system. Since then, many scholars at home and abroad have made more in-depth research on inverted pendulum system, and have made many substantive breakthroughs.

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2. Description of single inverted pendulum system

Vehicle-inverted pendulum system is the research object of various control theories. As long as the Car-inverted pendulum system is mentioned, it is generally considered that its mathematical model has been finalized. In fact, the mathematical model of Car-inverted pendulum is related to the drive system. The common model only corresponds to the case of DC motor. If the actuator is an AC servo motor, it is not the model. This paper mainly analyses the Car-inverted pendulum system driven by DC motor. Vehicle inverted pendulum system is a typical object to test the control mode. It is characterized by high order, instability, non-linearity and strong coupling. Only by adopting effective control mode can it be controlled stably.

After ignoring the air resistance and various friction, the linear inverted pendulum system can be abstracted into a system composed of sudden uniform rods of a car, as shown in the following figure 2:

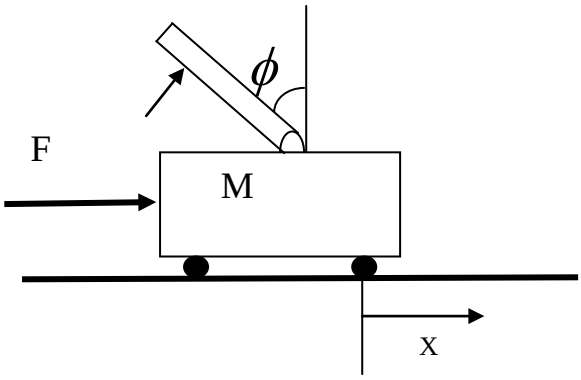

Fig.2 Simple model of linear inverted pendulum

F is the horizontal force applied to the car. X is the displacement of the car and the tilt angle of the pendulum. If no control force is exerted on the car, the inverted pendulum will tilt left or right. The purpose of control is to exert force on the car horizontally when the inverted pendulum has a deflection angle, and keep the inverted pendulum in a vertical position through the horizontal movement of the car. That is to say, the state parameters of the control system are used to keep the inverted pendulum stable.

Parameter value can see below:

M The Quality of the Car 1.096Kg
m Quality of Swing Rod 0.109Kg
B Friction of car 0.1N/m/sec
l The Length from the Rotary Axis of the Swing Rod to the Center of Mass of the Rod 0.2 5m
I Pendulum inertia 0.0034kg*m*m
F The force applied to the car
X Car position
θ The angle between the pendulum rod and the downward vertical direction
ϕ The angle between the pendulum rod and the vertical direction upward

3. Systematic Mathematical Modeling of Single Inverted Pendulum

The ultimate control aim of inverted pendulum system is to make inverted pendulum such an unstable controlled object. By introducing appropriate control strategies, inverted pendulum system can become a stable system that can meet various performance indicators. The control model of single inverted pendulum system is widely used at home and abroad at present, which is the basis of studying various control algorithms. Foundation.

The system consists of computer, motion control card, servo mechanism, inverted pendulum, body and photoelectric code disk. As shown in the figure, the photoelectric code disc feeds back the

displacement and velocity signals of the car to the servo driver and the motion control card, and the position of the pendulum rod. The speed signals are also fed back to the motion control card by the photoelectric code disc. The computer reads real-time data from the motion control card to determine the control decision (which direction the car moves, moving speed, acceleration, etc.). The control decision is realized, and the corresponding control quantity is generated, so that the motor drives the car and keeps the balance.

4. Structural parameters

The inverted pendulum is unstable. If there is no proper control force acting on it, it will fall in any direction at any time. In this paper, only two-dimensional problems are considered, that is, the inverted pendulum only moves in the plane shown in Figure 3. The control force F acts on the car. The pendulum length is l, the mass is m, the mass of the car is M, the instantaneous displacement of the car is x, and the instantaneous position of the pendulum is (x+2L*sin). Under the action of external forces, the system generates motion. It is assumed that the center of gravity of the pendulum rod lies in its geometric center. Set the input force F and the output angle as pendulum angle.

5. Equations of motion of the system

The control requirement is to adjust the position X of the car and keep the balance of the pendulum rod when the car is subjected to external force F.

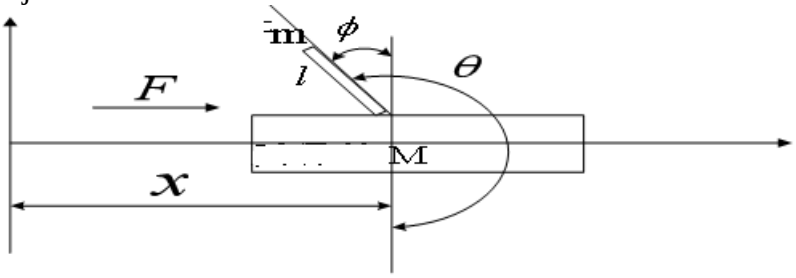

Fig.3 Force analysis diagram of linear inverted pendulum

The kinematics equation of the inverted pendulum system can be deduced by Newton mechanics as following:

$$\begin{cases} M\ddot{x} + N = F - b\dot{x} \\ N = m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta \\ P - mg = -ml\ddot{\theta}\sin\theta - ml\ddot{\theta}\cos\theta \\ -pl\sin\theta - Nl\cos\theta = I\ddot{\theta} \end{cases} \tag{1}$$

Where $\theta = \pi + \phi$, $\cos\theta = -\cos\phi$, $\sin\theta = -\sin\phi$.

By eliminating P and N, the following equation is obtained:

$$(M+m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F \tag{2}$$

$$(M+m)\ddot{x} + mgl\sin\theta = -ml\ddot{x}\cos\theta \tag{3}$$

Then the model of linear inverted pendulum was built, and it can be used in the control example for university students.

6. Summary

Firstly, the basic composition, physical structure and control logic of the inverted pendulum system are introduced. Through the analysis of physical parameters and mechanical forces of the inverted pendulum system, and according to Newton's law of mechanics, the kinematics model of the inverted pendulum system is established, which provides a model basis for subsequent control and simulation analysis. .

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